

A Dielectrically Loaded Converging Guide Accelerator

P. SPRANGLE

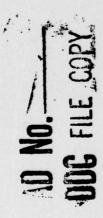
Electron Beam Applications Branch Plasma Physics Division



and

A. T. DROBOT

Science Applications Incorporated McLean, Virginia 22101



December 1977





NAVAL RESEARCH LABORATORY Washington, D.C.

Approved for public release; distribution unlimited.

(18) SBIE		
N 3 = 410 735 (14)		
9 AD-EDDD 135 14	L-MR-3660	/
	READ INSTRUCTIONS	1
REPORT DOCUMENTATION PAGE	BEFORE COMPLETING FORM	
NRL Memorandum Report 3660	RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle)	TYPE OF REPORT & PERIOD COVERED	
DIEL FORDIGALLY LOADED CONVERGING CHIDE	NRL problem.	
ACCELERATOR.	6. PERFORMING ORG. REPORT NUMBER	
AUTHOR(a)	8. CONTRACT OR GRANT NUMBER(*)	
Sprangle A. T/Drobot	1192	
Phillip	(12/1/p)	-
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK	
Naval Research Laboratory Washington, D. C. 20375	NRL Problem H02-4 RR Q1	109
, 0	Program Element 61153N	
11. CONTROLLING OFFICE NAME AND ADDRESS	December 77	
Office of Naval Research Arlington, Virginia 22217	13: HUMBER OF PROSE	
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)	18 15. SECURITY CLASS. (of this report)	
	INCI ACCIDID	
	UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
D. S.	SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report)		
Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, If different from	nm Report)	
17. DISTRIBUTION STATEMENT (OF THE SEPTECT STREETS IN BISER 25, IT STREETS		
18. SUPPLEMENTARY NOTES		
†Science Applications, Inc., McLean, Virginia 22101		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number. Ion accelerator		
Converging guide		
Dielectrically lined guide		
20. ABSTRACT (Continue on reverse side it necessary and identity by block number) A scheme for collectively accelerating ions trapped in a spi		
intense relativistic electron beam will be discussed. In this sch		
well of a low phase velocity space charge wave and accelerated velocity of the wave. One way of accelerating the wave is by p		
a converging waveguide. The converging structure has the effect		
energy and thus, accelerating the wave. This paper is concerne	d mainly with achieving low phase	
	(Continued)	1

DD 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

251 95\$

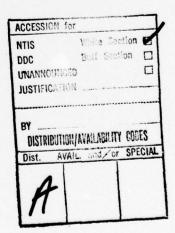
20. Abstract (Continued)

velocity space charge waves in a loaded waveguide. At the limiting electron beam current the minimum phase velocity of a space charge wave approaches zero. By lining the converging guide accelerator with an appropriate frequency dependent dielectric material, the initial phase velocity can be made small enough to trap low energy protons (\mathfrak{S}_5 MeV) at currents below the limiting current. The electric field associated with the accelerating wave can be as high as a MeV/cm.

Cappiox.

CONTENTS

I.	INTRODUCTION	1
II.	DIELECTRIC LOADED WAVEGUIDES	4
III.	DISCUSSION AND EXAMPLE	9
	REFERENCES	10



A DIELECTRICALLY LOADED CONVERGING GUIDE ACCELERATOR

I. INTRODUCTION

Attempts to generate high fluxes of very energetic ion beams have turned to collective acceleration schemes. During the past several years a number of new collective accelerators have been proposed, which in principle could generate ion currents orders of magnitude larger than conventional accelerators. The idea common to the collective accelerators is the introduction of an active medium in which a potential well is formed and gradually accelerated to high velocities carrying with it trapped ions. The use of non-neutral intense relativistic electron beams, IREBs, seems a natural choice for the accelerating medium. Space charge electric fields produced by IREBs are capable of reaching values which are a few times 10⁶ volts/cm. The advantage of using IREBs for producing fields of this magnitude is that they form absolute potential wells providing transverse and logitudinal trapping of ions. One class of collective acceleration schemes relies on the formation and acceleration of a single electrostatic potential well associated with the front of the intense electron beam. $^{1-13}$ By carefully controlling the velocity of the single potential well, ions trapped in the well can be accelerated.

Note: Manuscript submitted November 14, 1977.

Another class of collective accelerators relies on the regulation of the phase velocity of a wave containing trapped ions. The wave may be either a cycletron or space charge eigenmode supported on an IREB. The wave accelerator which utilizes the cyclotron mode is the Auto Resonant Accelerator, ARA. 14-17 In the ARA concept, an axially symmetric slow cycloron wave, initially having a low phase velocity, is accelerated by gradually decreasing the external axial magnetic field. Ions which are trapped in the low phase velocity region remain trapped and are accelerated if the magnetic field is properly tapered. Since the external magnetic field is decreasing as a function of axial position, the beam diameter increases, thus requiring the waveguide diameter to increase.

A more recently proposed collective wave accelerator, the Converging Guide Accelerator, CGA, is based on controlling the phase velocity of a space charge wave. $^{18-20}$ Large amplitude space charge waves can be grown on IREBs by propagating the beam through a slow wave structure. $^{21-22}$ In the simplest version of the CGA a space charge wave supported by an IREB propatating through a cylindrical waveguide is accelerated by adiabatically decreasing the waveguide radius. A strong uniform magnetic field along the axis of the converging waveguide prevents transverse electron motion. Since the energy of the electron beam increases as it propagates through the converging guide it becomes possible to regulate the phase velocity of an imposed large amplitude space charge wave. A possible difficult with the initial version of the CGA occurs at low phase velocities, $v_{\rm ph} > 0.3c$. In order to achieve low phase velocities at the input of the waveguide, electron beam currents close to the limiting current, as determined by the dimensions of the guide, are required. 23,24 Operating very close to the

limiting electron beam current may be difficult for a number of reasons:

i) the phase velocity is very sensitive to the electron beam current, ii) attempts to grow a space charge wave may result in the formation of a virtual cathode, and, iii) slight voltage variations on the diode might cause the beam current to exceed the limiting current. At higher initial phase velocities, corresponding to injected ion energies 40 MeV, operation near the limiting current is not necessary. Therefore, the original version of the CGA seems promissing as a second stage of an accelerator in which protons are injected at 40 MeV energies or higher.

As indicated, the phase velocity of the slow space charge wave approaches zero as the beam current approaches the limiting current. For a fixed radius waveguide, the limiting current can be increased by the insertion of a dielectric liner. However, even in a dielectrically loaded wave guide zero phase velocity space-charge waves occur only at the limiting current. There seems to be no obvious way to obtaining zero phase velocity space charge waves on beams propagating below the limiting current, without resorting to periodic wave guide structures. 25 However, we will show that a dielectric liner with a monotonically decreasing dielectric constant, as a function of frequency, can be used to effectively lower the phase velocity of the slow space charge wave. By lining the walls of the converging guide accelerator with a frequency dependent dielectric lower phase velocity space charge waves can be propagated on the electron beam. This modification of the CGA concept may significantly relax the initial energy required of the injected protons.

II. DIELECTRIC LOADED WAVEGUIDES

The waveguide-beam configuration is shown in Figure 1, the electron beam is infinitely thin, $\Delta x \rightarrow 0$, constrained to move only in the axial direction by a large magnetic field, $B_0\hat{e}_z$. The electron beam is symmetrically located between the conducting plates of the dielectrically loaded waveguide. This configuration is a model for a thin annular beam propagating through a dielectrically loaded cylindrical waveguide with a center conductor when the aspect ratio is large. The dielectric material is assumed to be anisotropic and frequency dependent. Since the electron beam is symmetrically located within the guide, only modes with axially symmetric logitudinal electric fields will be considered.

a) Limiting Current

The limitations on the electron beam current in a dielectrically loaded waveguide, as in an unloaded waveguide, arise from the self-electrostatic potential of the beam. Taking the potential at the conducting plates, $x = \pm x_2$, to be zero and assuming that the beam rise time is sufficiently long so that the D.C. dielectric constant ²⁶ can be used we find that the steady state electrostatic potential on the z-axis is

$$\phi(x = 0) = \frac{-4\pi |e| n_b \Delta x}{2} \quad \lambda(0)$$
 (1)

where n_b is the local beam density, $x_{21} = x_2 - x_1$, $\lambda(\omega) = x_2 - x_{21}(1 - \epsilon_x^{-1}(\omega))$ and $\epsilon_x(0) > 1$ is the D.C. dielectric constant which is assumed lossless. The potential drop from the edge of the beam, $x = \pm \Delta x/2$ to the center of the beam, x = 0, is

$$\frac{\Delta \phi}{\phi(x=0)} = \frac{\Delta x}{2} \lambda^{-1}(0) . \tag{2}$$

For a thin beam the potential depression across the beam can be very small compared to the potential at the beam. Therefore, a uniform density and velocity profile across the beam can be assumed. The current of the planar beam of width L along the y axis is given by:

$$I_{e} = \beta_{e} \left(\lambda_{inj} - \lambda_{e} \right) I_{0}$$
 (3)

where $I_e = |e|n_b c \beta_e L \Delta x$, $\beta_e = v_e/c$, $\lambda_e = (1-\beta_e^2)^{-1/2}$

$$I_0 = (m_0 c^2/|e|)(L/2\pi)\lambda^{-1}(0), m_0 c^2/|e| = 17 \times 10^3 \text{ A} \text{ and}$$

 $(\gamma_{ini}-1)m_0c^2$ is the kinetic energy of the electrons at the anode.

The maximum or limiting current is given by:

$$I_L = I_0 (\gamma_{inj}^{2/3} - 1)^{3/2}$$
 (4)

and occurs when $\gamma_e = \gamma_L \equiv (\gamma_{inj})^{1/3}$. The presence of the dielectric increases the maximum current that can be passed through a waveguide of fixed outer dimensions. In terms of the ratio $R \equiv I_e/I_L$, the local beam plasma frequency in a dielectrically loaded guide is given by

$$\omega_{\rm b}^2 = 2\gamma_{\rm e}^3 c^2 \beta_{\rm e}^2 \xi/(\Delta x \lambda(0))$$
 (5)

where $\omega_b = (4\pi |e|^2 n_b/m_0)^{1/2}$, $\xi = (\gamma_L \beta_L/(\gamma_e \beta_e))^3 R \le 1$ and $\beta_L = (1 - 1/\gamma_L^2)^{1/2}$.

b) Wave Characteristics

In an infinite axial magnetic field the linear beam current density is given by

$$J_z(z,t) = \frac{i\omega}{4\pi} \frac{\omega_b^2/\gamma_e^3}{(\omega - v_e k)^2} E_z(0,z,t).$$
 (6)

The symmetric axial electric field in the region $o \le |x| \le x_1$ is:

$$E_{z}(x,z,t) = E_{o} \left\{ \cosh(qx) - q \frac{\Delta x}{2} \left(\frac{P}{q} \right)^{2} \sinh(qx) \right\} \exp i(kz - \omega t)$$
 (7a)

and for $x_1 \le |x| \le x_2$, is given by

$$E_{z}(x,z,t) = E_{o} \left\{ \cosh(qx_{1}) - q \frac{\Delta x}{2} \left(\frac{P}{q}\right)^{2} \sinh(qx_{1}) \right\}$$

$$\times \left\{ \frac{\sinh(\tilde{q}(x_{2}-x))}{\sinh(\tilde{q}x_{21})} \right\} \exp i(kz-\omega t)$$
(7b)

where E_{Ω} is the amplitude of the axial electric field at the beam

$$\begin{split} & q = (k^2 - \omega^2/c^2)^{1/2}, \; p^2 = -q^2 \; \{1 - (\omega_b^2/\gamma_e^3)/(\omega - v_e k)^2\} \; , \\ & \text{and} \; \widetilde{q} = \sqrt{\epsilon_z}(\omega) \; (k^2/\epsilon_\chi(\omega) \; - \; \omega^2/c^2)^{1/2} \; . \end{split}$$

The components of the dielectric constant, ε_{χ} and ε_{Z} , are assumed to be frequency dependent. The remaining field components, E_{χ} and B_{γ} , can be obtained from the axial electric field. It is not difficult to show that the linear dispersion relation under the stated assumptions is:

$$(q/\tilde{q})\varepsilon_{z}(\omega) \coth (\tilde{q}x_{21}) = \tanh (qx_{1}) \frac{\left\{q \frac{\Delta x}{2} \left(\frac{P}{q}\right)^{2} \coth (qx_{1}) - 1\right\}}{\left\{1 - q \frac{\Delta x}{2} \left(\frac{P}{q}\right)^{2} \tanh (qx_{1})\right\}}$$
(8)

The exact characteristics of the slow space charge wave will depend on the form of $\varepsilon_{\rm X}(\omega)$ and $\varepsilon_{\rm Z}(\omega)$ as determined by the dielectric material. Since the slow space charge wave has low phase velocities when ${\rm qx}_{21} < 1$, for ω and k small, a small argument expansion of the dispersion relation in Eq. (8) is appropriate and yields,

$$(\omega - v_e k)^2 = \xi \beta_e^2 (c^2 k^2 \lambda(\omega) - \omega^2 x_2) / \lambda(0),$$
 (9)

where the expression for ω_b^2 in Eq. (5) was used. Solving (9) for the phase velocity of the slow space charge wave as a function of ω gives

$$\beta_{\text{ph}} = \frac{\beta_{\text{e}} (1 - \xi \lambda(\omega)/\lambda(0))}{1 + (\xi \lambda(\omega)/\lambda(0))^{1/2} \left\{ 1 - \beta_{\text{e}}^2 \frac{x_2}{\lambda(\omega)} \left(1 - \xi \lambda(\omega)/\lambda(0) \right)^{1/2} \right\}}$$
(10)

where $\beta_{ph} = \omega/(ck)$. For small frequencies and wavenumbers the phase velocity of the slow space charge wave is independent of $\varepsilon_{z}(\omega)$. When the dielectric constant is independent of frequency, we find that $\lambda(\omega)/\lambda(0) = 1$ and the expression for the phase velocity reduces to the form:

$$\beta_{\text{ph}} = \frac{\beta_{\text{e}}(1-\xi)}{1+\xi^{1/2} \left\{1-\beta_{\text{e}}^2 x_2/\lambda(0)(1-\xi)\right\}^{1/2}}$$
(11)

This shows that the zero phase velocity can only be achieved when $\xi=1$, i.e., at the limiting current, independent of the dielectric constant. At currents below I_L the presence of a frequency independent dielectric increases the phase velocity as compared to the phase velocity in the absence of the dielectric. We now observe from Eq. (10) that the phase velocity can be substantially reduced if the dielectric constant decreases monotonically with frequency. In this case $\varepsilon_\chi(0) > \varepsilon_\chi(\omega)$ and as a result $\lambda(\omega) > \lambda(0)$ reducing the phase velocity below the value obtained when the dielectric is frequency independent and below the value achievable in the absence of any dielectric.

It is advantageous to have the dielectric constant $\varepsilon(\omega)$ approach unity as a function of frequency rapidly to avoid stimulated Chernekov radiation when the beam mode intersects higher TM waveguide modes. We have also found that a low frequency Cherenkov instability can occur. To prevent low frequency Cherenkov instabilities we see from Eq. (10) that the following condition must be satisfied:

$$\frac{\lambda(\omega)}{\lambda(0)} \ge \frac{\beta_{e}^{2}}{\frac{\lambda(0)}{x_{2}} + \xi \beta_{e}^{2}} . \tag{12}$$

Since the dielectric constant has been chosen to be a montonically decreasing function of frequency it is sufficient to satisfy the inequality in (12) at ω = 0.

The maximum linear accelerating electric field is reached when the electrons of the beam begin to trap in the logitudinal wave; at this point linear theory breaks down. Electrons begin to trap when, in the wave frame, the potential of the wave equals the kinetic energy of the particles. In the laboratory frame this corresponds to the electric field

$$E_{z} = \frac{k}{\gamma_{ph}} (\gamma_{ew} - 1) \frac{m_{o}c^{2}}{|e|}$$
 (13)

where $\gamma_{ph}=(1-(\omega/ck)^2)^{-1/2}$ and $\gamma_{ew}=\gamma_e\gamma_{ph}(1-\beta_e\beta_{ph})$ is the relative gamma between the electron beam and the wave. Stated in another way, linear theory breaks down when the perturbed electron density; δn , approaches the ambient density n_b ; this occurs when the electric field has the value

$$E_z = \gamma_e^3 (\omega - v_e k)^2 \frac{m_o}{|e|k}$$
 (14)

The expressions in Eq. (13) and (14) yield roughly the same estimate for the maximum electric field. Substituting the dispersion relation of Eq. (9) into Eq. (14) we find that the ratio of the maximum accelerating field to the self electrostatic field at the surface of the beam, $E_{\rm self}$, is given by

$$|E_{z}/E_{self}| = (\lambda(\omega) - \beta_{ph}^{2} x_{2}) k \leq 1$$
where $E_{self} = R(\gamma_{inj}^{2/3} - 1)^{3/2} (\beta_{e}\lambda(0))^{-1} \frac{m_{o}c^{2}}{|e|}$. (15)

III. DISCUSSION AND EXAMPLE

The unloaded converging guide accelerator concept appears to be a viable scheme for accelerating protons from energies > 40 MeV to final energies of ∿ GeV. For initial proton energies below 40 MeV the relativistic electron beam must be operated very close to the limiting current which, as pointed out, may lead to practical problems. As a possible way of overcoming the need for high energy injected ions, a frequency dependent dielectrically loaded guide has been considered. By the appropriate choice of dielectric material the phase velocity of the negative energy space charge wave can be reduced to the point where protons at energies as low as 5 MeV can be picked up and accelerated.

As an illustration we choose a dielectric material with the property that $\varepsilon(0)$ = 10 and $\varepsilon(\omega)$ \rightarrow 1 for large frequencies, where ω is the frequency of the accelerating space charge wave. The parameters of the electron beam are γ_e = 2, R \equiv I $_e$ /I $_L$ = 0.8 and Δx = 0.5 cm. The electron beam current per unit length in the y direction is \sim 24 k A/cm. The dispersion curve of the slow space charge wave for these parameters is shown in Figure 2. Figure 3 shows the phase velocity and maximum accelerating electric field as a function of axial wavenumber. The actual choice of dielectric material with the appropriate frequency characteristics and electrical properties has not been decided upon. But it should be noted that many materials have the property that their dielectric constant is a monotonically decreasing function of frequency. For example, Ethyl alcohol has a dielectric constant 27 of ε = 24 at f = 10 c/sec. and decreases to ε = 1.7 at f = 10 c/sec.

REFERENCES

- N. Rostoker, in Proceedings of the Seventh International Conference on High Energy Accelerators (Publishing House of Academy of Sciences of Armenia, Yerevan, U.S.S.R., 1970), Vol. II, p. 509.
- J. R. Uglum, S. E. Graybill, and W. H. McNeill, Bull. Am. Phys. Soc. 14, 1047 (1969).
- S. E. Rosinskii, A. A. Rukhadze, and V. G. Rukhlin, Psi'ma
 Zh. Eksp. Teor. Fiz. 14, 53 (1971) JETP Lett. 14, 34 (1971).
- 4. J. W. Poukey and N. Rostoker, Plasma Phys. 13, 897 (1971).
- 5. S. D. Putnam, Phys. Rev. Lett. 25, 1129 (1970).
- J. M. Wachtel and B. J. Eastlund, Bull. Am. Phys. Soc. <u>14</u>, 1047 (1969).
- 7. Craig L. Olson, Phys. Fluids 18, 585 (1975).
- 8. Craig L. Olson, Part. Accel. 6, 197 (1975).
- R. J. Faehl, R. B. Miller and B. B. Godfrey, Bull. Am. Phys. Soc. 21, 1165 (1976).
- 10. R. B. Miller and David C. Straw, J. Appl. Phys. <u>48</u>, 1061 (1977).
- C. N. Boyer, W. W. Destler and H. Kim, IEEE Trans on Nuc. Sci. 24, 1625 (1977).
- R. B. Miller, R. J. Faehl, T. C. Genoni and W. A. Proctor,
 IEEE Trans. on Nuc. Sci. <u>24</u>, 1648 (1977).
- R. K. Parker, J. A. Pasour, W. O. Doggett, D. Pershing and
 R. L. Gullickson, Bull. Am. Phys. Soc. Atlanta, (1977).
- 14. M. L. Sloan and W. E. Drummond, Phys. Rev. Lett. 31, 1234 (1975).

- 15. M. L. Sloan and W. E. Drummond, in Proceedings of the Ninth International Conference on High Energy Accelerators, Stanford Accelerator Center, Stanford, California, 1974, CONF-740, 522 (National Technical Infomration Service, Springfield, Va., 1974), p. 283.
- 16. B. Godfrey, IEEE Trans. Plas. Sci. to be published.
- 17. R. J. Faehl, B. B. Godfrey, B. F. Newberger, W. R. Shanahan and L. E. Thode, IEEE Trans. on Nuc. Sci. 24, 1637 (1977).
- 18. P. Sprangle, A. T. Drobot and W. M. Manheimer, Bull. Am. Phys. Soc. 20, 1353 (1975)
- P. Sprangle, A. T. Drobot and W. M. Manheimer, Phys. Rev. Lett.
 36, 1180 (1976).
- 20. S. V. Yadavalli, Appl. Phys. Lett. 29, 272 (1976).
- G. Gammel, J. A. Nation and M. E. Read, IEEE, International Conference on Plasma Science, Conf. Record, 172 (1977).
- G. Gammel, J. A. Nation and M. E. Read, Bull. of Am. Phys. Soc.
 1184 (1976).
- 23. R. J. Briggs, Phys. Fluids 19, 1257 (1976)
- 24. B. B. Godfrey, IEEE Trans. on Plas. Sci. to be published.
- 25. This concept is currently being investigated by the Authors.
- 26. By a D. C. dielectric constant we mean the value at frequencies comparable to $1/\tau_r$, where τ_r is the beam rise time
- 27. A. Von Hippel, Dielectric Materials and Applications, The Technology Press of M.I.T. and John Wiley and Sons, Inc., New York, 1958, pp. 362.

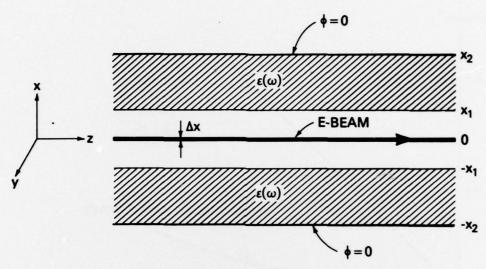


Fig. 1 — Configuration of the thin beam, dielectrically lined waveguide model

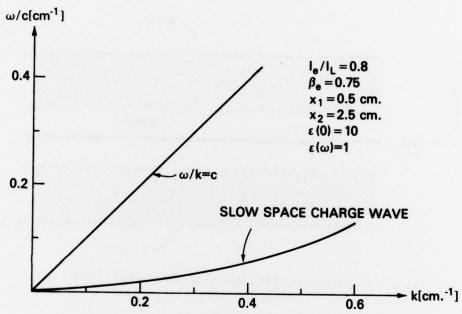


Fig. 2 — Dispersion curve of the slow space charge wave in the dielectrically lined waveguide

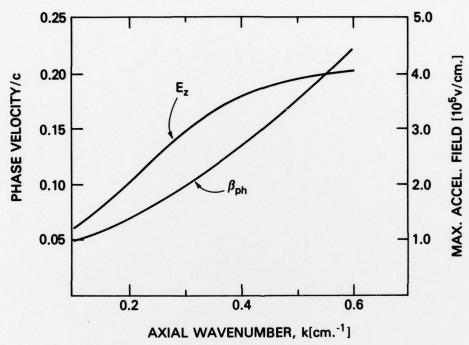


Fig. 3 — Phase velocity and maximum electric field associated with the slow space charge wave